Bellman-Ford Algm

⇒ It solves the Single Source shortest paths problem in the general case in which edge weights may be negative. The algm will also detect if there are any negative weight cycles (such that there is no solution).

Analysis

1. Initialization takes $O(V)$ time
2. Loop $|V|-1$ times through all edges checking the relaxation condition to compute minimum distances (line 5 to 9)

$$(|V|-1)\, O(E) = O(VE)$$

3. Loop through all edges checking for negative weight cycles which occurs if any of the relaxation conditions fail — $O(E)$

Therefore the running time of Bellman-Ford algm is $O(V + VE + E)$

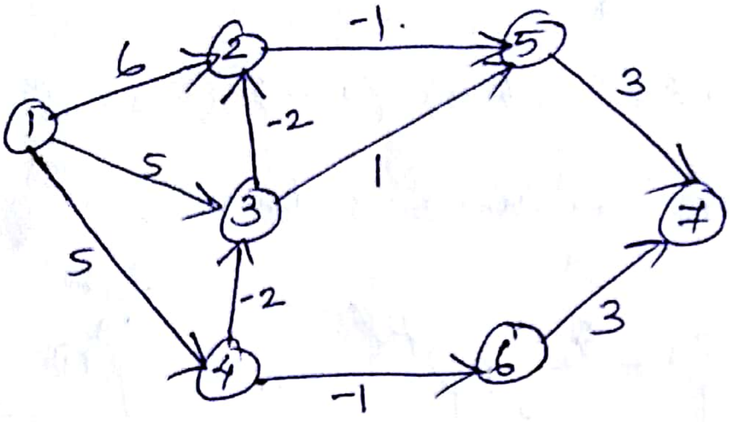
$$= \underline{O(VE)}$$

Bellman-Ford algm.

```
Bellman-Ford (G, w, s)
for each vertex v ∈ V[G]
        do d[v] ← ∞
           π[v] ← NIL
d[s] ← 0
for i ← 1 to |V[G]| - 1
        do for each edge (u,v) ∈ E[G]
              do if d[v] > d[u] + w(u,v)
                    then d[v] = d[u] + w(u,v)
                         π[v] ← u
for each edge (u,v) ∈ E(G)
        do if d[v] > d[u] + w(u,v)
              then return FALSE
return TRUE
```

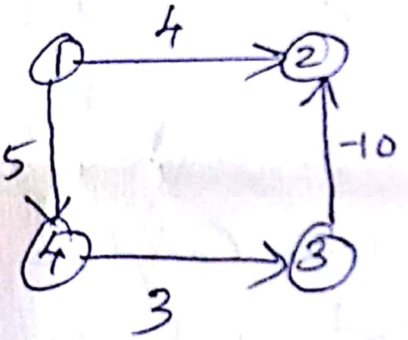
Bellman - Ford

eg



do it as homework

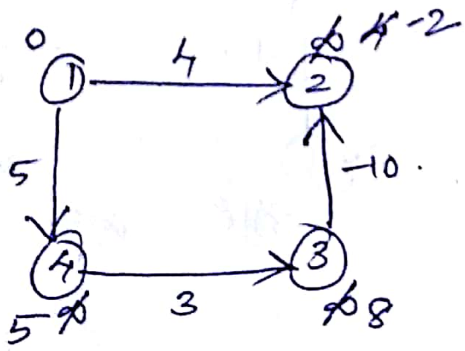
working eg



edges order → (3,2), (4,3), (1,4) (1,2).

relaxes is $n-1 = 4-1 = 3$ times.

start at 1.



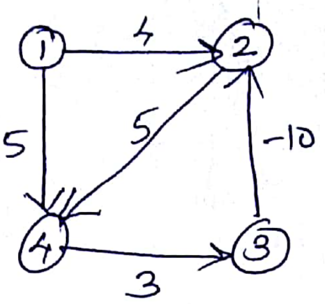
	1	2	3	4
Initially	0	∞	∞	∞
1st pass	0	4	~~∞~~	~~∞~~ 5
2nd pass	0	4	8	5
3rd pass	$\boxed{0}$	$\boxed{-2}$	$\boxed{8}$	$\boxed{5}$

shortest distance

→ if relax one more time, there is no change in the values.

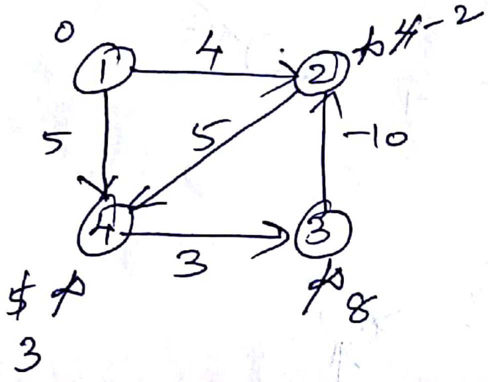
Drawback of Bellman-Ford.

Let's Consider the given graph.



edges → (3,2) (4,3) (1,4) (1,2) (2,4)

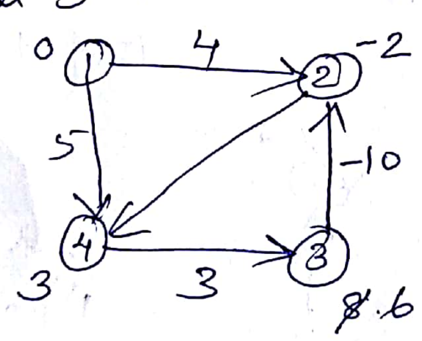
relax — (n-1) = 3 times.



start ↓	1	2	3	4
initially	0	∞	∞	∞
1st pass	0	4	∞	5
2nd	0	4	8	5
3rd	0	-2	8	3.

→ ~~there~~ now we Complete our (n-1) times relaxation, but just do ~~one~~. one more relaxation. rather one here

→ After 3rd pass.



	1	2	3	4
After 3rd pass	0	-2	8	3.
if do one more pass of relaxations	0	-2	6	3

Value changes

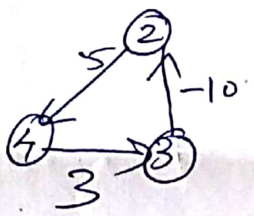
→. Now, one more vertex has just relaxed or value changes, that should not happen.

→ After Completing the (n-1) time pass, we should get the answer. If it repeat for one ~~time~~ more time ~~relaxation~~ i.e. relax the edges one more time, the values should not change.

→ here if we repeat one more time, the values will again change.

→ So there will be a problem. why got this pblm. Bellman-Ford is unable to solve it.

→ Reason is, there is a negative ~~edge~~ weight cycle:



this is a cycle, if we add the edge weights, then $5+3+-10 = -2$ we get a negative weight

→ So in this way, we will be going on, ~~the~~ getting the path reduced ~~on~~ every time. → there is no end, not getting the answer

→ So if there is a negative weight cycle, the graph cannot be solved.

→ Bellman-Ford Algm fails to solve the graph with negative weight cycle.

→ But Bellman-Ford algm can detect if there is a negative weight cycle or not. Because after performing $(n-1)$ time pass, do one more time relaxation and check whether there is a change is ~~the~~ any values. If any one value ~~is~~ getting change, that means there is a negative weight cycle.