MODULE V

REFERENCE:

DISCRETE MATHEMATICAL STRUCTURES WITH APPLICATIONS TO COMPUTER SCIENCE

-J.P. TREMBLAY & R. MANOHAR

TOPICS

- Propositional Logic :
  - Propositions
  - Logical Connectives
  - Truth tables
  - Tautologies and Contradictions
  - Contra positive
  - Logical Equivalences and Implications
- Rules of Inference :
  - Validity of arguments

Propositional Logic

Propositional Logic is the logic of compound statements built from simpler statements using so-called Boolean connectives.

Some applications in computer science:

- Design of digital electronic circuits.
- Expressing conditions in programs.
- Queries to databases & search engines.

Propositions

- A *proposition* is a statement that is either true (T) or false (F).
- A proposition (statement) may be denoted by a variable like P, Q, R,…, called a proposition (statement) variable.

Propositions

Examples

- Propositions:

1. I am a man.
2. I am taller than 170 cm.
3. You are studying in Baptist U.
4. 1 +1 = 3 .

- Not propositions:

1. How are you?
2. Go to catch the dog.
3. I like this color.

Truth Table

- A *truth table* displays the relationships between the truth values of propositions.

- Truth tables are especially valuable in the determination of the truth values of propositions constructed from simpler propositions.

Logical Operators (Connectives)

Sr. No.	Connective	Symbol	Compound statement
1	AND	^	Conjunction
2	OR	∨	Disjunction
3	NOT	¬	Negation
4	XOR	⊕	Exclusive-OR
5	If....then	→	Conditional or implication
6	If and only if (iff)	↔	Biconditional

- **Conjunction**

Let p and q be propositions.

The proposition "p and q", denoted by $p \wedge q$, is the proposition that is true when both p and q are true and is false otherwise.

The proposition $p \wedge q$ is called the ***conjunction*** of p and q.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- Translate into symbolic form of the statement

  Jack and Jill went up the hill

  - P : Jack went up the hill
  - Q: Jill went up the hill

- Statement can be written as P^Q

Disjunction

The proposition "*p* or *q*", denoted by $p \vee q$, is the proposition that is false when *p* and *q* are both false and true otherwise. The proposition $p \vee q$ is called the ***disjunction*** of *p* and *q*.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation of a Proposition

Let p be a proposition. The statement

"It is not the case that p"

is another proposition, called the *negation of p*. The negation of p is denoted by $\neg p$ and read "not p".

p	$\neg p$
T	F
F	T

Example

P : "It is a sunny day."

$\neg p$: "It is not the case that it is a sunny day.", or simply "It is not a sunny day."

- **Exclusive Or**

Let p and q be propositions.

The ***exclusive or*** of p and q, denoted by true when exactly one of p and q is true and is false otherwise.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Construct a truth table for P ∨ ¬Q

P	**Q**	**¬Q**	**P ∨ ¬Q**
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

Conditional Propositions

- **Implication**

Let p and q be propositions.

The *implication* $p \rightarrow q$

is the proposition that is false when p is true and q is false and true otherwise.

- In this implication, p is called the *hypothesis* and q is called the *conclusion*.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Remarks:

- Equivalent expressions of implication

1. if p, then q
2. p is sufficient for q
3. p implies q
4. p only if q
5. q is necessary for p

- Related Implications

1. $q \rightarrow p$ is called the converse of $p \rightarrow q$
2. $\neg q \rightarrow \neg p$ is called the contrapositive of $p \rightarrow q$

- **Biconditional**

  Let p and q be propositions. The *biconditional* $p \leftrightarrow q$ is the proposition that is true when p and q have the same truth values and is false otherwise.

  - In this biconditional, p is necessary and sufficient for q, or p if and only if q.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- **Converse:**

If $P \rightarrow Q$ is an implication then $Q \rightarrow P$ is called the converse of $P \rightarrow Q$

- **Contra positive :**

If $P \rightarrow Q$ is an implication then the implication $\neg Q \rightarrow \neg P$ is called it's contra positive.

- **Inverse:**

If $P \rightarrow Q$ is a an implication then $\neg P \rightarrow \neg Q$ is called its inverse.

Example 6:

Let P: You are good in Mathematics.
Q: You are good in Logic.
Then, $P \rightarrow Q$: If you are good in Mathematics then you are good in Logic.

1) Converse: $(Q \rightarrow P)$
If you are good in Logic then you are good in Mathematics.

2) Contra positive: $\neg Q \rightarrow \neg P$
If you are not good in Logic then you are not good in Mathematics.

3) Inverse: $(\neg P \rightarrow \neg Q)$
If you are not good in Mathematics then you are not good in Logic.

Tautology, Contradiction and Contingency

- A compound proposition that is always true, no matter what the truth values of the propositions that occur in it, is called a ***tautology***.

- A compound proposition that is always false is called a ***contradiction***.

- A proposition that is neither a tautology nor a contradiction is called a ***contingency***.

- Truth table of examples of a tautology and a contradiction

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Logically Equivalent

- The propositions p and q are called logically equivalent if $p \leftrightarrow q$ is a tautology.
- The notation $p \Leftrightarrow q$ denotes that p and q are logically equivalent.

- The following truth table shows that the compound propositions logically equivalent.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

- Complete the following truth table to show that $\neg p \vee q$ and $p \rightarrow q$ are logically equivalent.

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T			
T	F			
F	T			
F	F			

Logical Equivalences

Equivalence	Name
$p \wedge T \Leftrightarrow p$ $p \vee F \Leftrightarrow p$	Identity laws
$p \vee T \Leftrightarrow T$ $p \wedge F \Leftrightarrow F$	Domination laws
$p \wedge p \Leftrightarrow p$ $p \vee p \Leftrightarrow p$	Idempotent laws
$\neg(\neg p) \Leftrightarrow p$	Double negative law
$p \vee q \Leftrightarrow q \vee p$ $p \wedge q \Leftrightarrow q \wedge p$	Commutative laws
$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$ $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$	De Morgan's laws
$p \rightarrow q \Leftrightarrow \neg p \vee q$	
Double Negative	$\neg (\neg p) \Longleftrightarrow p$
Absorption	$p \vee (p \wedge q) \Longleftrightarrow p$ $p \wedge (p \vee q) \Longleftrightarrow p$

Logical Equivalence Involving Implications :

- Let P & Q be two statements.
- The following table displays some useful equivalences for implications involving conditional and biconditional statements.

Sr. No.	Logical Equivalence involving implications
1	$P \to Q \equiv \neg P \vee Q$
2	$P \to Q \equiv \neg Q \to \neg P$
3	$P \vee Q \equiv \neg P \to Q$
4	$P \wedge Q \equiv \neg(P \to \neg Q)$
5	$\neg(P \to Q) \equiv P \wedge \neg Q$
6	$(P \to Q) \wedge (P \to r) \equiv P \to (Q \wedge r)$
7	$(P \to r) \wedge (Q \to r) \equiv (P \vee Q) \to r$ $P \vee \neg Q$
8	$(P \to Q) \vee (P \to r) \equiv P \to (Q \vee r)$
9	$(P \to r) \vee (Q \to r) \equiv (P \wedge Q) r$
10	$P \leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$
11	$P \leftrightarrow Q \equiv \neg P \leftrightarrow \neg Q$
12	$P \leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$
13	$\neg(P \leftrightarrow Q) \equiv P \leftrightarrow \neg Q$

Well ordered Formulas

- A compound statement obtained from statement letters by using one or more connectives is called a statement pattern or statement form.

- Thus, if P, Q, R, … are the statements (which can be treated as variables) then any statement involving these statements and the logical connectives ¬, ∧, ∨, →, ↔ is a statement form or a well ordered formula or statement pattern.

- Any statement involving propositional variable and logical connectives is a well formed formula

RULES OF INFERENCE

- Rule P: A premise may be introduced at any point in the derivation
- Rule T : A formula S may be introduced in the derivation if S is tautologically implied by any one or more of the preceding formulas in the derivation.

Table 1-4.3 EQUIVALENCES

E_1	$\neg\neg P \Leftrightarrow P$	(double negation)
E_2	$P \wedge Q \Leftrightarrow Q \wedge P$	(commutative laws)
E_3	$P \vee Q \Leftrightarrow Q \vee P$	
E_4	$(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$	(associative laws)
E_5	$(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$	
E_6	$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$	(distributive laws)
E_7	$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$	
E_8	$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$	(De Morgan's laws)
E_9	$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$	
E_{10}	$P \vee P \Leftrightarrow P$	
E_{11}	$P \wedge P \Leftrightarrow P$	
E_{12}	$R \vee (P \wedge \neg P) \Leftrightarrow R$	
E_{13}	$R \wedge (P \vee \neg P) \Leftrightarrow R$	
E_{14}	$R \vee (P \vee \neg P) \Leftrightarrow \mathbf{T}$	
E_{15}	$R \wedge (P \wedge \neg P) \Leftrightarrow \mathbf{F}$	
E_{16}	$P \rightarrow Q \Leftrightarrow \neg P \vee Q$	
E_{17}	$\neg(P \rightarrow Q) \Leftrightarrow P \wedge \neg Q$	
E_{18}	$P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$	
E_{19}	$P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$	
E_{20}	$\neg(P \rightleftarrows Q) \Leftrightarrow P \rightleftarrows \neg Q$	
E_{21}	$P \rightleftarrows Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$	
E_{22}	$(P \rightleftarrows Q) \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$	

Table 1-4.2 IMPLICATIONS

I_1	$P \wedge Q \Rightarrow P$	(simplification)
I_2	$P \wedge Q \Rightarrow Q$	
I_3	$P \Rightarrow P \vee Q$	(addition)
I_4	$Q \Rightarrow P \vee Q$	
I_5	$\neg P \Rightarrow P \to Q$	
I_6	$Q \Rightarrow P \to Q$	
I_7	$\neg(P \to Q) \Rightarrow P$	
I_8	$\neg(P \to Q) \Rightarrow \neg Q$	
I_9	$P, Q \Rightarrow P \wedge Q$	
I_{10}	$\neg P, P \vee Q \Rightarrow Q$	(disjunctive syllogism)
I_{11}	$P, P \to Q \Rightarrow Q$	(modus ponens)
I_{12}	$\neg Q, P \to Q \Rightarrow \neg P$	(modus tollens)
I_{13}	$P \to Q, Q \to R \Rightarrow P \to R$	(hypothetical syllogism)
I_{14}	$P \vee Q, P \to R, Q \to R \Rightarrow R$	(dilemma)

Demonstratethat R is a valid inference from the premises $P \rightarrow Q, Q \rightarrow R$ and P

Solution

1) $P \rightarrow Q$	Rule P
2) P	Rule P
3) Q	Rule T,(1)(2) and Modus Ponens
4) $Q \rightarrow R$	Rule P
5) R	Rule T,(3,4) and Modus Ponens

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Let p and q be propositions.

The proposition "p and q", denoted by $p \wedge q$, is the proposition that is true when both p and q are true and is false otherwise.

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F	F	F

- Translate into symbolic form of the statement

  Jack and Jill went up the hill

    - P : Jack went up the hill
    - Q: Jill went up the hill

- Statement can be written as $P \wedge Q$

- **Disjunction**

The proposition "p or q", denoted by $p \vee q$, is the proposition that is false when p and q are both false and true otherwise. The proposition $p \vee q$ is called the ***disjunction*** of p and q.

p	q	$p \vee q$
T	T	T
T	F	T
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Negation of a Proposition

Let p be a proposition. The statement

"It is not the case that p"

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Example

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- **Exclusive Or**

Let p and q be propositions.

The ***exclusive or*** of p and q, denoted by true when exactly one of p and q is true and is false otherwise.

p	q	$p \oplus q$
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T	F	T
F	T	T
F	F	F

Construct a truth table for P ∨ ¬Q

P	Q	¬Q	P ∨ ¬Q
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

Conditional Propositions

- **Implication**

Let p and q be propositions.

The *implication* $p \rightarrow q$

is the proposition that is false when p is true and q is false and true otherwise.

- In this implication, p is called the *hypothesis* and q is called the *conclusion*.

p	q	$p \rightarrow q$
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T	F	F
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F	F	T

Remarks:

- Equivalent expressions of implication

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2. p is sufficient for q
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- Related Implications

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- **Biconditional**

  Let p and q be propositions. The *biconditional* $p \leftrightarrow q$ is the proposition that is true when p and q have the same truth values and is false otherwise.

  - In this biconditional, p is necessary and sufficient for q, or p if and only if q.

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F	F	T

- **Converse:**

If $P \rightarrow Q$ is an implication then $Q \rightarrow P$ is called the converse of $P \rightarrow Q$

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Example 6:

Let P: You are good in Mathematics.
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- Truth table of examples of a tautology and a contradiction

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
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Logically Equivalent

- The propositions p and q are called logically equivalent if $p \leftrightarrow q$ is a tautology.
- The notation $p \Leftrightarrow q$ denotes that p and q are logically equivalent.

- The following truth table shows that the compound propositions logically equivalent.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

- Complete the following truth table to show that $\neg p \lor q$ and $p \to q$ are logically equivalent.

p	q	$\neg p$	$\neg p \lor q$	$p \to q$
T	T			
T	F			
F	T			
F	F			

Logical Equivalences

Equivalence	Name
$p \wedge T \Leftrightarrow p$ $p \vee F \Leftrightarrow p$	Identity laws
$p \vee T \Leftrightarrow T$ $p \wedge F \Leftrightarrow F$	Domination laws
$p \wedge p \Leftrightarrow p$ $p \vee p \Leftrightarrow p$	Idempotent laws
$\neg(\neg p) \Leftrightarrow p$	Double negative law
$p \vee q \Leftrightarrow q \vee p$ $p \wedge q \Leftrightarrow q \wedge p$	Commutative laws
$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$ $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$	De Morgan's laws
$p \rightarrow q \Leftrightarrow \neg p \vee q$	
Double Negative	$\neg (\neg p) \Longleftrightarrow p$
Absorption	$p \vee (p \wedge q) \Longleftrightarrow p$ $p \wedge (p \vee q) \Longleftrightarrow p$

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5	$\neg(P \rightarrow Q) \equiv P \wedge \neg Q$
6	$(P \rightarrow Q) \wedge (P \rightarrow r) \equiv P \rightarrow (Q \wedge r)$
7	$(P \rightarrow r) \wedge (Q \rightarrow r) \equiv (P \vee Q) \rightarrow r$ $P \vee \neg Q$
8	$(P \rightarrow Q) \vee (P \rightarrow r) \equiv P \rightarrow (Q \vee r)$
9	$(P \rightarrow r) \vee (Q \rightarrow r) \equiv (P \wedge Q) r$
10	$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$
11	$P \leftrightarrow Q \equiv \neg P \leftrightarrow \neg Q$
12	$P \leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$
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Well ordered Formulas

- A compound statement obtained from statement letters by using one or more connectives is called a statement pattern or statement form.

- Thus, if P, Q, R, … are the statements (which can be treated as variables) then any statement involving these statements and the logical connectives $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$ is a statement form or a well ordered formula or statement pattern.

- Any statement involving propositional variable and logical connectives is a well formed formula

RULES OF INFERENCE

- Rule P: A premise may be introduced at any point in the derivation

- Rule T : A formula S may be introduced in the derivation if S is tautologically implied by any one or more of the preceding formulas in the derivation.

Table 1-4.3 EQUIVALENCES

E_1	$\neg\neg P \Leftrightarrow P$	(double negation)
E_2	$P \wedge Q \Leftrightarrow Q \wedge P$	(commutative laws)
E_3	$P \vee Q \Leftrightarrow Q \vee P$	
E_4	$(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$	(associative laws)
E_5	$(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$	
E_6	$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$	(distributive laws)
E_7	$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$	
E_8	$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$	(De Morgan's laws)
E_9	$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$	
E_{10}	$P \vee P \Leftrightarrow P$	
E_{11}	$P \wedge P \Leftrightarrow P$	
E_{12}	$R \vee (P \wedge \neg P) \Leftrightarrow R$	
E_{13}	$R \wedge (P \vee \neg P) \Leftrightarrow R$	
E_{14}	$R \vee (P \vee \neg P) \Leftrightarrow \mathbf{T}$	
E_{15}	$R \wedge (P \wedge \neg P) \Leftrightarrow \mathbf{F}$	
E_{16}	$P \rightarrow Q \Leftrightarrow \neg P \vee Q$	
E_{17}	$\neg(P \rightarrow Q) \Leftrightarrow P \wedge \neg Q$	
E_{18}	$P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$	
E_{19}	$P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$	
E_{20}	$\neg(P \rightleftarrows Q) \Leftrightarrow P \rightleftarrows \neg Q$	
E_{21}	$P \rightleftarrows Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$	
E_{22}	$(P \rightleftarrows Q) \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$	

Table 1-4.2 IMPLICATIONS

I_1	$P \wedge Q \Rightarrow P$	(simplification)
I_2	$P \wedge Q \Rightarrow Q$	(simplification)
I_3	$P \Rightarrow P \vee Q$	(addition)
I_4	$Q \Rightarrow P \vee Q$	(addition)
I_5	$\neg P \Rightarrow P \rightarrow Q$	
I_6	$Q \Rightarrow P \rightarrow Q$	
I_7	$\neg(P \rightarrow Q) \Rightarrow P$	
I_8	$\neg(P \rightarrow Q) \Rightarrow \neg Q$	
I_9	$P, Q \Rightarrow P \wedge Q$	
I_{10}	$\neg P, P \vee Q \Rightarrow Q$	(disjunctive syllogism)
I_{11}	$P, P \rightarrow Q \Rightarrow Q$	(modus ponens)
I_{12}	$\neg Q, P \rightarrow Q \Rightarrow \neg P$	(modus tollens)
I_{13}	$P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$	(hypothetical syllogism)
I_{14}	$P \vee Q, P \rightarrow R, Q \rightarrow R \Rightarrow R$	(dilemma)

Demonstratethat R is a valid inference from the premises $P \rightarrow Q$, $Q \rightarrow R$ and P

Solution

1) $P \rightarrow Q$	Rule P
2) P	Rule P
3) Q	Rule T,(1)(2) and Modus Ponens
4) $Q \rightarrow R$	Rule P
5) R	Rule T,(3,4) and Modus Ponens